

### **The Biggest Product**

You are given a number  $N$ , and you can split  $N$  into any set of natural numbers whose sum is  $N$  (natural numbers are 1, 2, 3, ...). Your goal is to find the set with the biggest product.

For example, suppose  $N$  is 10. Here are some choices you could make:

- Leave  $N$  as it is. Your “product” is 10.
- Split  $N$  into  $\{5, 2, 2, 1\}$ . Your product is  $5 \times 2 \times 2 \times 1 = 20$
- Split  $N$  into  $\{7, 3\}$ . Your product is  $7 \times 3 = 21$

Find the biggest product for any natural number  $N$ . Prove your results, i.e. give a convincing argument that your patterns continue, your equations/expressions are correct, etc.

Start working on this problem now. If you need some guidance, see below (but try starting without it; it will be more satisfying if you figure these things out on your own):

#### **Guidance:**

- Start with small  $N$ 's. Make an organized table and look for patterns.
- Try to articulate any rules you discover along the way and to justify them. For example, do any of your biggest products include a 1? Explain why or why not. Do any of your biggest products include a 7? Why or why not?
- See if you can find a way to describe the biggest product so that you can tell someone else how to find the biggest product for a large number, say 200, and if you can come up with a convincing justification that you've found the biggest product.