

The Handshake Problem
Debra K. Borkovitz
Wheelock College
[Terms of Use](#)

For this assignment, you will start by working on a problem and then read and critique several different students' approaches to the same problem. Be prepared to share your work next class.

The problem:

There are 20 people in a room. If everyone shakes hands with everyone else, how many handshakes will take place?

Start by working on the problem. Can you convince a skeptic that your answer is correct? Can you generalize? What if there were 100 people in the room?

STOP HERE. Try different strategies. Look for good representations. Justify your reasoning.

When you think you have solved the problem, or a portion of the problem, read on.

Now let's look at some students' strategies for doing this problem. As you read each strategy, ask yourself the following questions:

Do I understand what the student is trying to do?

Does the student's strategy work for this problem?

Is this student's reasoning correct? Has the student justified his or her thinking?

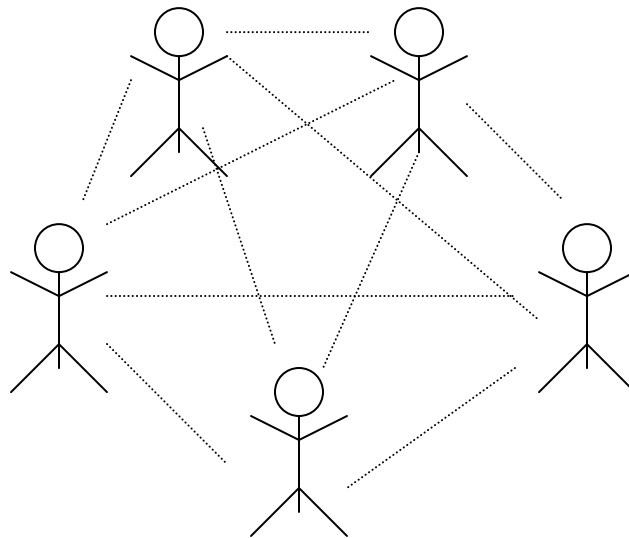
If the student made a mistake, can I fix it?

Can I take something from this student's work that will help me with another problem?

Manuel: I think this problem might be kind of easy. There are 20 people in the room. Each person shakes hands with everyone but himself or herself. So each of the twenty people shakes 19 hands. So it there would be 20 times 19 or 380 handshakes.

Stop and answer the above questions for Manuel.

Beatriz: Well I decided to look at a smaller case. I thought what if there were five people. Then I drew the people and drew lines connecting them.



I counted the lines and there were 10. So then I thought what if there was another person there. I thought that this person would shake 5 people's hands, so there would be a total of 15 handshakes -- the 10 ones from the old people and the 5 ones from this new person. Then I thought what if there was another person? I started getting confused, so I made a table:

People Handshakes

5	10	
		> +5 I put this here because the new person adds 5 handshakes
6	15	
		> +6 The new person has to shake hands with all 6 of the other people
7	21	
		> +7 And the pattern keeps going; the next one will be +8, etc.
8	28	
9	36	
10	45	
11	55	
12	66	
13	78	
14	91	
15	105	
16	130	
17	146	
18	163	
19	181	
20	200	

I understand how my pattern works, but I don't know how I could use it if the numbers went up to 1000 or something big like that.

Stop and answer the questions for Beatriz.

Carmel: My way is kind of like Beatriz's way except I started right at the beginning. With 2 people there is one handshake. Then with a third person you have to add 2 more handshakes. With a fourth person you have to add 3 more handshakes. The twentieth person shakes 19 hands, so you have to add the numbers from 1 to 19. I found a shortcut for adding up all the numbers. I grouped them in pairs that add up to 19 (except I left 19 by itself):

$$\begin{array}{cccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \frac{19}{19} & \frac{+18}{19} & \frac{+17}{19} & \frac{+16}{19} & \frac{+15}{19} & \frac{+14}{19} & \frac{+13}{19} & \frac{+12}{19} & \frac{+11}{19} & \frac{+10}{19} \end{array}$$

So adding the whole numbers from 1 to 19 is the same as adding ten 19's or the answer is $10 \times 19 = 190$.

Answer the questions for Carmel.

Katia: I made a chart, and put an "X" to represent a handshake. If there are 6 people, here's my chart (I call the people A, B, C, D, E, and F):

	A	B	C	D	E	F
A		X	X	X	X	X
B			X	X	X	X
C				X	X	X
D					X	X
E						X

I didn't put F in on the left because F already shook everyone's hand. I didn't put an "X" on the diagonals because people don't shake their own hands. Now here's what I noticed. There are 30 boxes on my chart and I filled in half of them. So I was thinking that with 20 people I would make a chart that would have 19 rows and 20 columns, so it would have $20 \times 19 = 380$ boxes, and half of them, or 190 would have X's in them, so there would be 190 handshakes. My way is kind of like Manuel's, except that Manuel forgot that if A shakes B's hand, then you can't also count B shaking A's hand.

Answer the questions for Katia.